

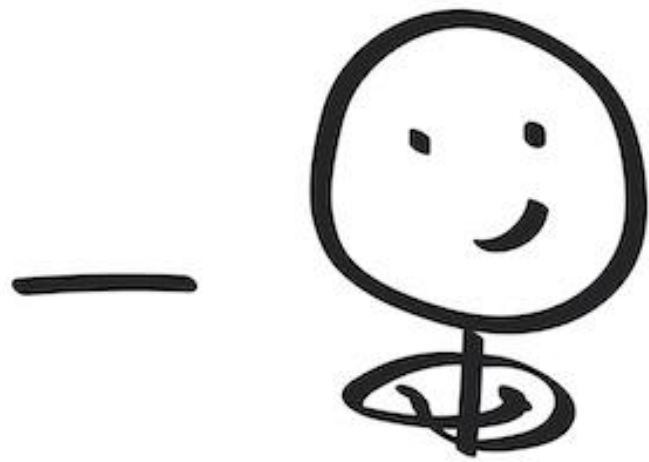
# Grade 11 S – Physics

## Chapter 13: Capacitor

# PROBLEM SOLVING



problem



thinking



solution

## Exercise 1

A parallel plate capacitor of capacitance  $C_1 = 48\text{nF}$  consists of two identical parallel conducting plates of common area  $S = 1000\text{cm}^2$ . The two plates are separated by a distance  $d = 0.1\text{mm}$ . An insulating material of relative permittivity  $\epsilon_r$  is used to separate the two plates. The following table gives the values of  $\epsilon_r$  of some mediums: Given  $\epsilon_0 = 8.85 \times 10^{-12}\text{F/m}$

Medium	Vacuum	<i>Paraffin</i>	<i>Mica</i>	<i>Porcelain</i>
$\epsilon_r$	1	4.5	5.4	6.5

1. Calculate the value of  $\epsilon_r$ . Deduce the nature of the insulating medium.
2. The capacitor is being charged under a voltage of  $U=24\text{V}$ .
  - a. Calculate the quantity of the electric charge  $q$  on the capacitor  $C_1$ .
  - b. Calculate the electric energy stored by the capacitor when it is completely charged.

## Exercise 1

$$C_1 = 48 \text{ nF}; S = 1000 \text{ cm}^2; d = 0.1 \text{ mm}; \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

Medium	Vacuum	Paraffin	Mica	Porcelain
$\varepsilon_r$	1	4.5	5.4	6.5

1. Calculate the value of  $\varepsilon_r$ . Deduce the nature of the insulating medium.

$$C = \varepsilon_0 \varepsilon_r \frac{S}{d} \Rightarrow 48 \times 10^{-9} = 8.85 \times 10^{-12} \varepsilon_r \frac{1000 \times 10^{-4}}{0.1 \times 10^{-3}}$$

$\Rightarrow 48 \times 10^{-9} = 8.85 \times 10^{-9} \varepsilon_r$  Referring to the table insulating medium is **Mica**

$$\Rightarrow \varepsilon_r = 5.4$$



## Exercise 1

$$C_1 = 48\text{nF}; S = 1000\text{cm}^2; d = 0.1\text{mm}; \varepsilon_0 = 8.85 \times 10^{-12}\text{F/m}$$

1. The capacitor is being charged under a voltage of  $U=24\text{V}$ .

a. Calculate the quantity of the electric charge  $q$  on the capacitor  $C_1$ .

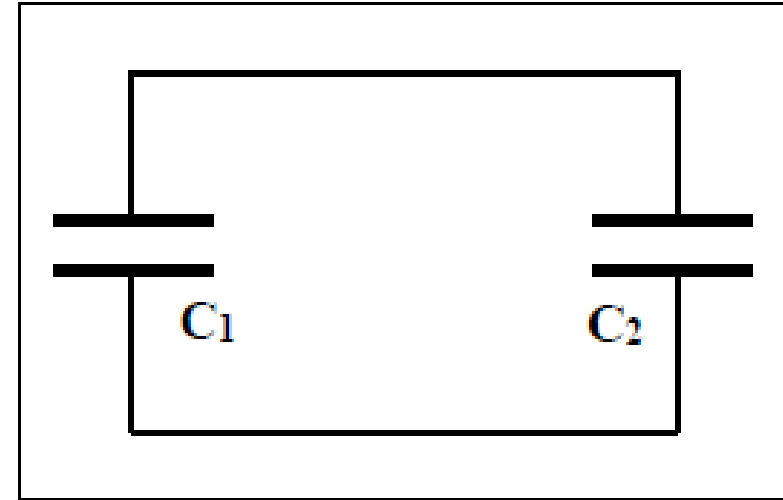
$$q = C \times U \quad \Rightarrow \quad q = 48 \times 10^{-9} \times 24 \quad \Rightarrow \quad q = 1.152 \times 10^{-6}\text{C}$$

b. Calculate the electric energy stored by the capacitor when it is completely charged.

$$w = \frac{1}{2}CU^2 \quad \Rightarrow \quad w = 0.5 \times 48 \times 10^{-9} \times 24^2$$
$$\Rightarrow \quad w = 1.3824 \times 10^{-5}\text{J}$$

## Exercise 1

3. The capacitor  $C_1$  being completely charged is now connected across another uncharged one of capacitance  $C_2 = 4C_1$  as indicated in the adjacent figure. At electric equilibrium, the electric charges on  $C_1$  and  $C_2$  are respectively  $q_1$  and  $q_2$ .



3.1) Write a relation between  $q_1$ ,  $q_2$  and  $q$ .

3.2) Deduce the voltage  $U'$  across each capacitor at electric equilibrium and the charges  $q_1$  and  $q_2$ .

## Exercise 1

$C_1 = 48\text{nF}$ ;  $C_2 = 192\text{nF}$ (uncharged); equilibrium,  $C_1$  have  $q_1$  and  $C_2$  have  $q_2$ .

3.1) Write a relation between  $q_1$ ,  $q_2$  and  $q$ .

$$Q_{\text{total initial}} = Q_{\text{total final}} \Rightarrow q + 0 = q_1 + q_2 \Rightarrow \mathbf{q = q_1 + q_2}$$

3.2) Deduce the voltage  $U'$  across each capacitor at electric equilibrium and the charges  $q_1$  and  $q_2$ .

$$\begin{aligned} q + 0 &= q_1 + q_2 \\ q &= C_1 \times U' + C_2 \times U' \Rightarrow 1.152 \times 10^{-6} = U'(48 + 192) \times 10^{-9} \\ q &= U'(C_1 + C_2) \Rightarrow \mathbf{U' = 4.8V} \end{aligned}$$

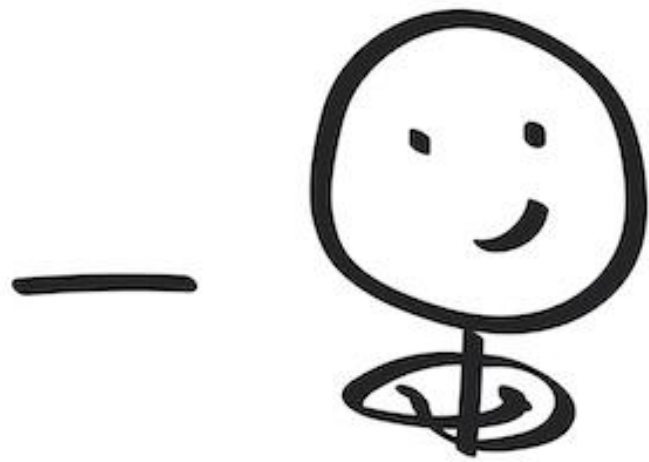




# PROBLEM SOLVING



problem



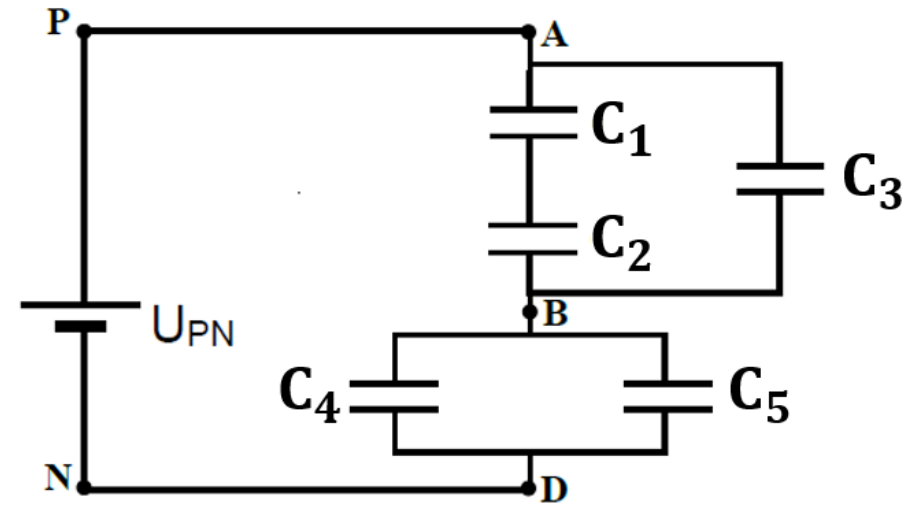
thinking



solution

## Exercise 2

Consider five capacitors of capacitance  $C_1 = C_2 = 2\mu F$ ,  $C_3 = 2\mu F$  and  $C_4 = C_5 = 1.5\mu F$  are connected to a dry cell of voltage  $U_{PN}$  as shown in the adjacent figure. Given that:  $U_{BD} = 5V$ .



- 1) Calculate the equivalent capacitance  $C_{eq}$  between A and D.
- 2) Calculate the charge  $Q_{BD}$ . Deduce the charge  $Q_{eq}$  of  $C_{eq}$ .
- 3) Deduce the voltage  $U_{PN}$ .
- 4) Calculate the charge and the voltage across each capacitor.

## Exercise 2

$C_1 = C_2 = 2\mu F$ ,  $C_3 = 2\mu F$  and  $C_4 = C_5 = 1.5\mu F$ ;  $U_{BD} = 5V$ .

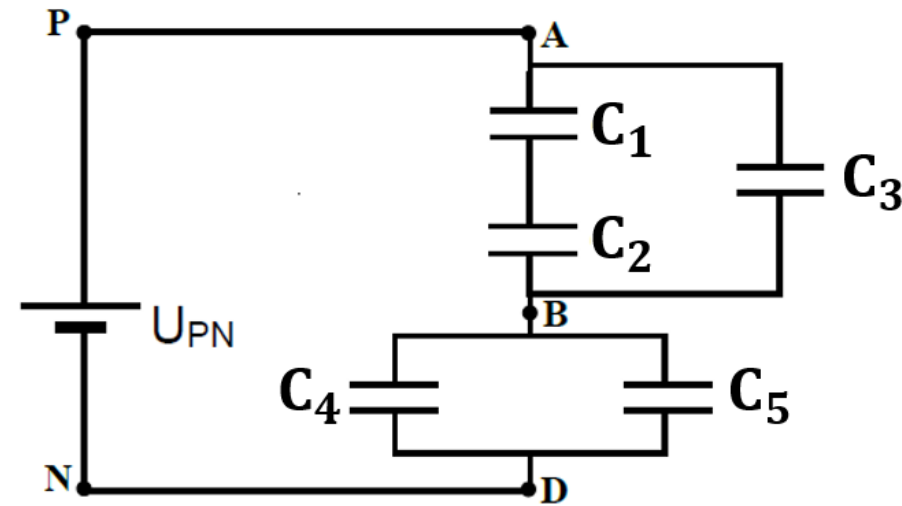
1) Calculate the equivalent capacitance  $C_{eq}$  between A and D.

$C_1$  and  $C_2$  are in series:  $C_{1,2} = \frac{C_1 \times C_2}{C_1 + C_2}$

$$\rightarrow C_{1,2} = \frac{2 \times 2}{2 + 2} = \frac{4}{4} = 1\mu F$$

$C_{1,2}$  and  $C_3$  are in parallel:  $C_{1,2,3} = C_{1,2} + C_3$

$$C_{1,2,3} = 1 + 2 = 3\mu F$$



## Exercise 2

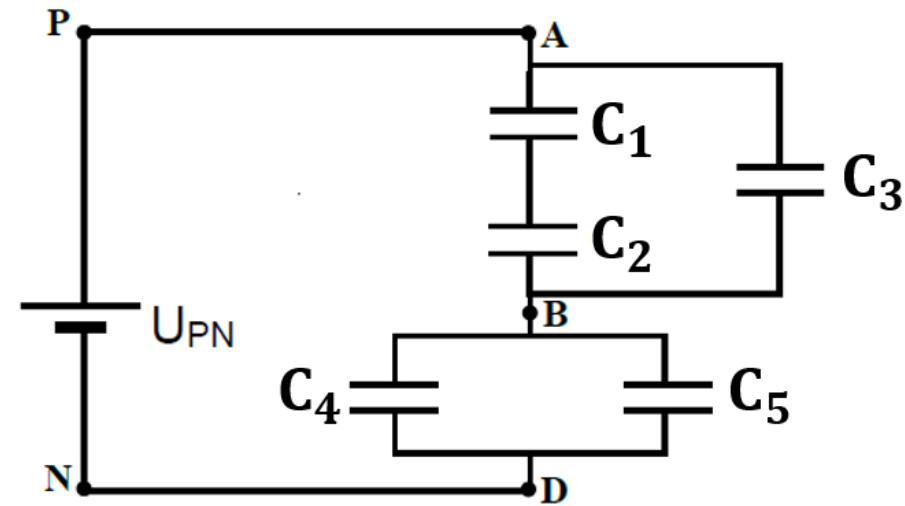
$C_1 = C_2 = 2\mu F$ ,  $C_3 = 2\mu F$  and  $C_4 = C_5 = 1.5\mu F$ ;  $U_{BD} = 5V$ .

$C_4$  and  $C_5$  are in parallel:  $C_{4,5} = C_4 + C_5$

$$\Rightarrow C_{4,5} = 1.5 + 1.5 \Rightarrow C_{4,5} = 3\mu F$$

$C_{1,2,3}$  and  $C_{4,5}$  are in series:

$$C_{eq} = \frac{C_{1,2,3} \times C_{4,5}}{C_{1,2,3} + C_{4,5}} = \frac{3 \times 3}{3 + 3} = \frac{9}{6} \Rightarrow C_{1,2,3} = 1.5\mu F$$





## Exercise 2

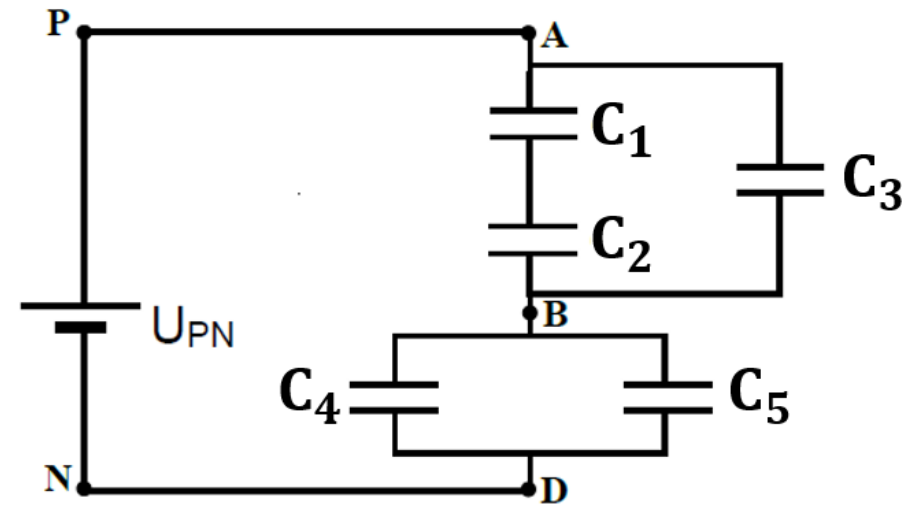
$C_1 = C_2 = 2\mu F$ ,  $C_3 = 2\mu F$  and  $C_4 = C_5 = 1.5\mu F$ ;  $U_{BD} = 5V$ .

2) Calculate the charge  $Q_{BD}$ . Deduce the charge  $Q_{eq}$  of  $C_{eq}$ .

$$Q_{BD} = C_{4,5} \times U_{BD} \Rightarrow Q_{BD} = 3nF \times 5$$

$$\Rightarrow Q_{BD} = 15\mu C$$

$$Q_{BD} = Q_{AB} = Q_{eq} = 15\mu C \quad \text{Law of uniqueness of charges in series}$$



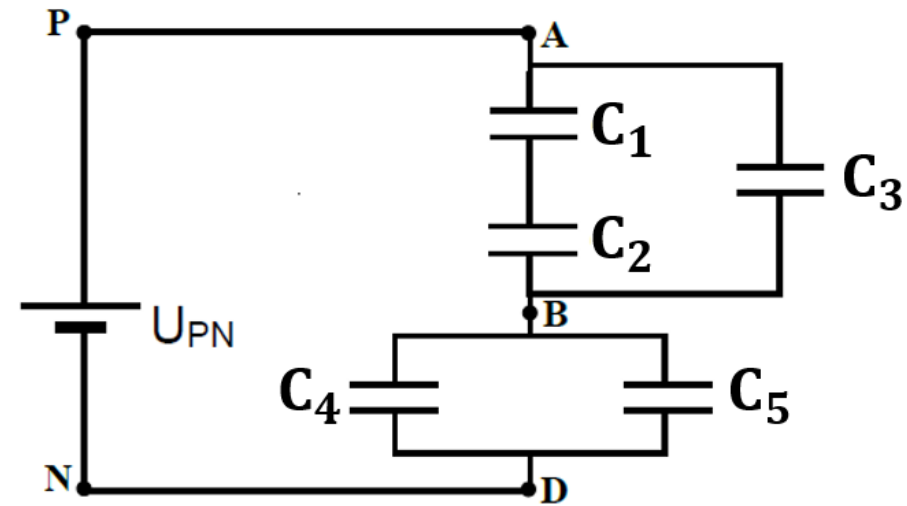
## Exercise 2

$C_1 = C_2 = 2\mu F$ ,  $C_3 = 2\mu F$  and  $C_4 = C_5 = 1.5\mu F$ ;  $U_{BD} = 5V$ .

3) Deduce the voltage  $U_{PN}$ .

$$Q_{eq} = C_{eq} \times U_{PN} \Rightarrow U_{PN} = \frac{Q_{eq}}{C_{eq}}$$

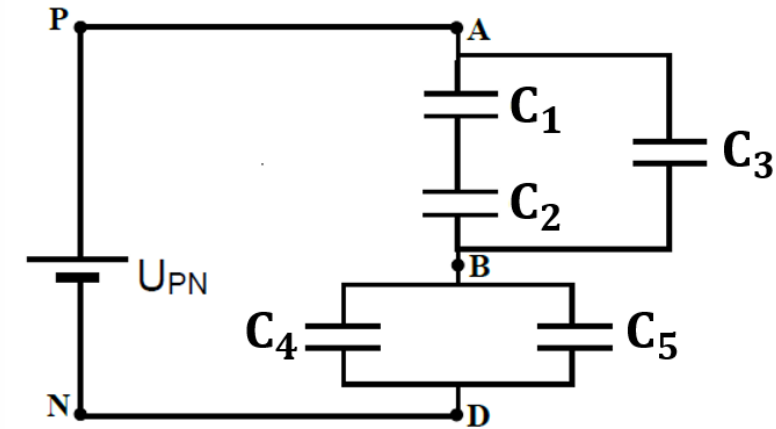
$$\Rightarrow U_{PN} = \frac{15 \times 10^{-6}}{1.5 \times 10^{-6}} \Rightarrow U_{PN} = 10V$$



## Exercise 2

$C_1 = C_2 = 2\mu F$ ,  $C_3 = 2\mu F$  and  $C_4 = C_5 = 1.5\mu F$ ;  $U_{BD} = 5V$ .

4) Calculate the charge and the voltage across each capacitor.



$C_4$  and  $C_5$  are in parallel then:  $U_4 = U_5 = U_{BD} = 5V$

$$Q_4 = C_4 \times U_4$$

$$\Rightarrow Q_4 = 1.5\mu F \times 5V$$

$$\Rightarrow Q_4 = 7.5\mu C$$

$$Q_5 = C_5 \times U_5 = 1.5\mu F \times 5V = 7.5\mu C$$

$$\Rightarrow Q_5 = 1.5\mu F \times 5V$$

$$\Rightarrow Q_5 = 7.5\mu C$$

## Exercise 2

$C_1 = C_2 = 2\mu F$ ,  $C_3 = 2\mu F$  and  $C_4 = C_5 = 1.5\mu F$ ;  $U_{BD} = 5V$ .

$C_1, 2$  and  $C_3$  are in parallel then:

$$U_{1,2} = U_3 = U_{AB} = 5V$$

$$Q_3 = C_3 \times U_3$$

$$\Rightarrow Q_3 = 2\mu F \times 5V$$

$$\Rightarrow Q_3 = 7.5\mu C$$

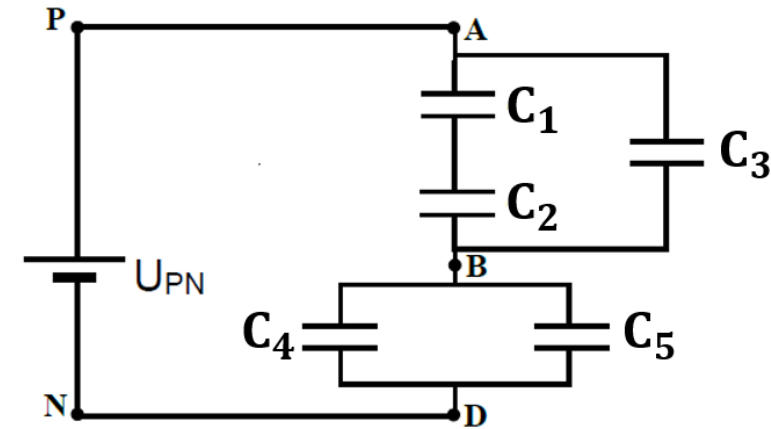
$C_1$  and  $C_2$  are in series then:

$$Q_{1,2} = C_{1,2} \times U_{1,2} = 1\mu F \times 5V = 5\mu C$$

$$\Rightarrow Q_1 = Q_2 = Q_{1,2} = 5\mu C$$

$$\Rightarrow U_1 = \frac{Q_1}{C_1} = \frac{5\mu C}{2\mu F} = 2.5V.$$

$$\Rightarrow U_2 = \frac{Q_2}{C_2} = \frac{5\mu C}{2\mu F} = 2.5V.$$





# The End

